Modular Arithemtee :-

$$\begin{array}{r|l} & 9 \\ \hline 11 & 101 \\ & 99 \\ \hline & 2 \rightarrow \text{remainder} \end{array}$$

$101 \bmod 11 = 2$

Ex) $\frac{13}{5} = 2$ remainder 3

$13 \bmod 5 = 3$

[1] Addition Formula

$a +_m b = (a+b) \bmod m$ ← small

$\boxed{a +_m b = (a+b) \bmod m}$

[2] Multiplication Formula

$a \cdot_m b = (a.b) \bmod m$ ← small

Ex) find $7 +_{11} 9$ and $7 \cdot_{11} 9$

Sol.

① $7 +_{11} 9 = (7+9) \bmod 11 = 16 \bmod 11 = \underline{5}$

$$\begin{array}{r|l} & 1 \\ \hline 11 & 16 \\ & 11 \\ \hline & 5 \end{array}$$

② $7 \cdot_{11} 9 = (7.9) \bmod 11 = 63 \bmod 11 = \underline{8}$

$$\begin{array}{r|l} & 5 \\ \hline 11 & 63 \\ & 55 \\ \hline & 8 \leftarrow \text{remainde} \end{array}$$

* Conversion between Binary, Octal, Hexadecimal.

~~[1] Decimal numbers~~

Decimal	Binary	octal	Hexadecimal
0	0	0	0
1	1	1	1
2	10	2	2
3	11	3	3
4	100	4	4
5	101	5	5
6	110	6	6
7	111	7	7
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

$\boxed{\text{هذا موجود بالكتاب ص 249}}$

$(241)_{10} = (11110001)_2$

Q.5: $(12345)_{10}$ Find Octal Expansion?
divide by 8

Sol. $12345 = 8 \cdot 1543 + 1$

$1543 = 8 \cdot 192 + 7$

$192 = 8 \cdot 24 + 0$

$24 = 8 \cdot 3 + 0$

$3 = 8 \cdot 0 + 3$

bottom-to-Top

$(12345)_{10} = (30071)_8$

Q.6: Find Hexadecimal expansion of decimal?
$(177130)_{10}$

Sol. $177130 = 16 \cdot 11070 + 10$

$11070 = 16 \cdot 691 + 14$

$691 = 16 \cdot 43 + 3$

$43 = 16 \cdot 2 + 11$

$2 = 16 \cdot 0 + 2$

$(177130)_{10} = (2B3EA)_{16}$

* $D \to B$, $D \to O$, $D \to H$

* binary $B \to D$ (decimal)
octal $O \to D$
H (Hexadecimal) $\to D$

Q.1: $(10101\ 1111)_2$ what's the decimal expansion ??

Sol.

$$(10101\ 1111)_2 = 1.2^8 + 0.2^7 + 1.2^6 + 0.2^5 + 1.2^4 + 1.2^3 + 1.2^2 + 1.2^1 + 1.2^0$$
$$= (351)_{10}$$

Q.2: $(7016)_8$ what's the Expansion decimal?

$$(7016)_8 = 7.8^3 + 0.8^2 + 1.8^1 + 6.8^0 = 3598_{10}$$

Q.3: $(2AE0B)_{16}$ convert into decimal number?

Sol.

$$(2AE0B)_{16} = 2.16^4 + 10.16^3 + 14.16^2 + 0.16^1 + 11.16^0$$
$$= 175\ 627$$

Q.4: $(241)_{10}$ convert into Binary expansion?

$(241)_{10} = (1111\ 0001)_2$

Sol.

bottom to top

$241 = 2.120 + 1$

$120 = 2.60 + 0$

$60 = 2.30 + 0$

$30 = 2.15 + 0$

$15 = 2.7 + 1$

$7 = 2.3 + 1$

$3 = 2.1 + 1$

$1 = 2.0 + 1$

Prime FactoriZation

2 is Prime number

Q.1: What's The Prime factorization of 56

56

2 Prime

28 composite (factor it)

2 Prime

14 composite (factor it)

2 Prime

7 Prime

the Prime factorization of 56 is

$2 \cdot 2 \cdot 2 \cdot 7 = 2^3 \cdot 7$

Q.2: Find Prime factorization of the following

60

2 Prime

30 composite

2 Prime

15 composite

3 Prime

5 Prime

105

5

21

3

7

the Prime factoriZation of 60 is $2 \cdot 2 \cdot 3 \cdot 5$

The Prime factoriZation of 105 is $5 \cdot 3 \cdot 7$

The G.C.F = 3.5 ⟸ GCD(60,105)

$60 = 2^2 . 3 . 5$
$105 = 3 . 5 . 7$

= <u>15</u>

GCD(60,105) → GCF(60,105)

Q. what's GCF of

$14p^4q$ $35pq^3$

GCF = $7pq$

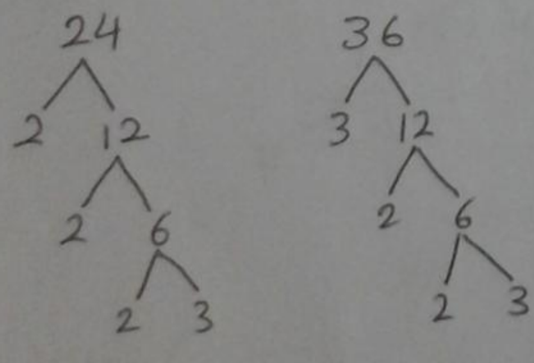
---

* <u>Prime number</u>:- is number having exactly two factors, <u>one</u> & <u>itself</u>. <u>Ex</u> 2, 11

---

* <u>Composite number</u>: is number which have two or more factors. <u>ex</u> 24, 12

<u>Ex</u> What is the Greatest Common Divisor of <u>24</u> and <u>36</u>?



$24 = 2.2.2.3 = 2^3 . 3$

$36 = 2.2.3.3 = 2^2 . 3^2$

$GCD = 2^2 . 3 = 4 . 3 = \underline{12}$

أصغر أس

So; GCD(24,36) = 12

Math 150 - week 9 - Ch 5 [1]

* Mathematical Induction :-

There are two Steps:-

[1] Basic Step: we have $P(1)$ is true.

[2] Inductive Step: if $P(k)$ is True, the $P(k+1)$ is also True.

Q.1 use the mathematical Induction to show that

$$1+2+3+\dots+n = \frac{n(n+1)}{2}$$

Sol. Basic step: first, we show $P(1)$ is true

L.H.S $1 = 1$

R.H.S $\frac{1(1+1)}{2} = \frac{2}{2} = 1$

it means $P(1)$ is true.

Inductive step:

we assume that $P(k)$ is true.

$$1+2+3+\dots+k = \frac{k(k+1)}{2}$$

and we have to show that $P(k+1)$ is true

$$1+2+3+\cdots+k+(k+1) = \frac{k(k+1)}{2} + (k+1)$$

$$= \frac{k(k+1)+2(k+1)}{2}$$

$$= \frac{(k+1)[k+2]}{2}$$

which is the $(k+1)^{th}$ term.

Q.2 use mathematicall Induction to show that $1^2+2^2+3^2+\cdots+n^2 = \frac{n(n+1)(2n+1)}{6}$

Sol. Basic Step:

L.H.S $1^2 = 1$

R.H.S $\frac{1(1+1)(2\times1+1)}{6} = \frac{6}{6} = 1$

P(1) is true

Inductive Step: assume that p(k) is true

$$1^2+2^2+3^2+\cdots+k^2 = \frac{k(k+1)(2k+1)}{6}$$

and we have to show that p(k+1) is true

$$1^2+2^2+3^2+\dots+k^2+(k+1)^2=\frac{k(k+1)(2k+1)}{6}+(k+1)^2$$

$$=\frac{k(k+1)(2k+1)+6(k+1)^2}{6}$$

$$=\frac{(k+1)[k(2k+1)+6(k+1)]}{6}$$

$$=\frac{(k+1)[2k^2+k+6k+6]}{6}$$

$$=\frac{(k+1)[2k^2+7k+6]}{6}$$

$$=\frac{(k+1)[(k+2)(2k+3)]}{6}$$

$(k+1)^{th}$ term

Q3 use the mathematical Induction to show that $1^3+2^3+3^3+\dots+n^3=\dfrac{n^2(n+1)^2}{4}$

Sol. Basic step.

L.H.S $1^3=1$

R.H.S $\dfrac{1^2(1+1)^2}{4}=\dfrac{4}{4}=1$

P(1) is true

Inductive step: assume that $P(k)$ is true

$$1^3+2^3+3^3+\dots+k^3=\frac{k^2(k+1)^2}{4}$$

and we have to show that $P(k+1)$ is true.

$$1^3+2^3+3^3+\dots+k^3+(k+1)^3=\frac{k^2(k+1)^2}{4}+(k+1)^3$$

$$=\frac{k^2(k+1)^2+4(k+1)^3}{4}$$

$$=\frac{(k+1)^2\left[k^2+4(k+1)\right]}{4}$$

$$=\frac{(k+1)^2\left[k^2+4k+4\right]}{4}$$

$$\boxed{\begin{array}{l} k^2+4k+4 \\ (k+2)(k+2) \\ (k+2)^2 \end{array}}$$

$$=\frac{(k+1)^2\,(k+2)^2}{4}$$

$(k+1)^{th}$ term.

Q.4 $f(0) = 3$

$f(n+1) = 2f(n) + 3$ find $f(1), f(2), f(3), f(4)$.

Sol. $n = 0$ $f(0+1) = 2f(0) + 3 \Rightarrow f(1) = 2 \cdot 3 + 3$

$\Rightarrow f(1) = 6 + 3 \Rightarrow \underline{\underline{f(1) = 9}}$

$n = 1$ $f(1+1) = 2f(1) + 3$

$f(2) = 2 \cdot 9 + 3$

$f(2) = 18 + 3$

$\underline{\underline{f(2) = 21}}$

$n = 2$ $f(2+1) = 2f(2) + 3$

$f(3) = 2 \cdot 21 + 3$

$f(3) = 42 + 3$

$\underline{\underline{f(3) = 45}}$

$n = 3$ $f(3+1) = 2f(3) + 3$

$f(4) = 2 \cdot 45 + 3$

$f(4) = 90 + 3$

$\underline{\underline{f(4) = 93}}$

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MATH 150 - week 10 - ch06 - Counting

* Product rule: we done the task in n_1 ways.

" " " second task in n_2 ways.

, Then There are $n_1 . n_2$ ways to do the Procedure.

Product
ضرب

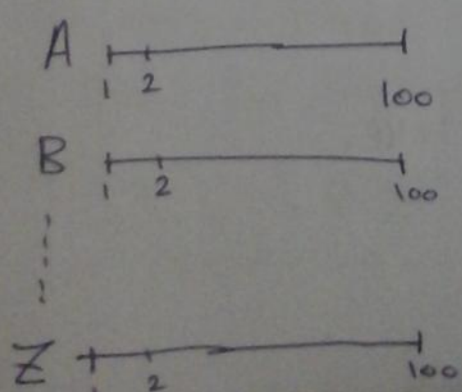
Ex1 Two Employees in a Company, ~~Add~~ Abdullah and Mohammed, and a building has 12 offices, How many different ways to assign different offices to there two employee.

Sol. For Abdullah to assign the rooms in 12 ways

for Mohammed " " " " " in 11 ways

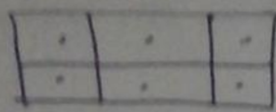
Total $12 . 11 = 132$ ways to assign offices there two employee.

Ex2



Sol. $26 . 100 = 2600$

* Pigin hole Principle:-



pigin 7

If k positive integer and k+1 are more objects are places into k boxes, then There is at least one box containing two or more objects.

Q.1/ 366 People

at least 2 People have the same birthday.

Sol. this statement is True

$365 + 1 = 366$ there is at least 2 People same birthday.

Q2/ 27 English words.

at least 2 that began the same letter.

Sol. This statement is True.

* Permutation and Combination

A permutation is an arrangment of objects in specific order.

In combination the position or order is not important.

① $nP_r = \dfrac{n!}{(n-r)!}$ where $nP_0 = 1$, $nP_1 = n$, $nP_n = n!$

② Combination

$${}^nC_r = \binom{n}{r} = \frac{nP_r}{r!} = \frac{n!}{r!\,(n-r)!}$$

where, ${}^nC_0 = {}^nC_n = 1$, ${}^nC_1 = {}^nC_{n-1} = n$

$1!$ or

$2! = 2\times1$

$3! = 3\times2\times1$

$4! = 4\times3\times2\times1$

⋮

Q.1/ Compute $6P_2$

Sol. $\dfrac{6!}{(6-2)!} = \dfrac{6!}{4!} = \dfrac{6\times5\times\cancel{4\times3\times2\times1}}{\cancel{4\times3\times2\times1}}$

$= 6\times5 = 30$

Q2/ How many diffferent 3 digits number can be make from the digit 4, 5, 6, 7, 8 if the digit can appear just once.

use Permutation

Sol. $^5P_3 = \frac{5!}{(5-3)!} = \frac{5!}{2!} = \frac{5\times4\times3\times2\times1}{2\times1} = \underline{\underline{60}}$

Q.3/ Evalute $^7C_2$

Sol.
$$^7C_2 = \frac{7!}{2!(7-2)!} = \frac{7!}{2!\,5!} = \frac{7\times6\times5\times4\times3\times2\times1}{2\times1\times5\times4\times3\times2\times1}$$
$$= \frac{42}{2} = \underline{\underline{21}}$$

Q.4/ five People are in a club, and 3 are going to be in the "planning commity" How many differenA ways this commity can be

Sol. $^5C_3 = \frac{5!}{3!(5-3)!} = \underline{\underline{10}}$

Q.5/ 14 Juoneuors students.
23 seneors students.
4 sent to be in the conference.

① 4 Students $\Leftarrow$ How many diffievent ways select 4 students?

Sol. $^{37}C_4 =$ $14+23=\underline{37}$ Students

$$\rightarrow \frac{37!}{4!(37-4)!} = 66045$$

(b) 2 juneors out of 14

2 seneons out of 23

Sol. $^2C_{14} \times {}^2C_{23} = \frac{14!}{2!(14-2)!} \times \frac{23!}{2!(23-2)!}$

$= 23023$

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* Binomial theorm

$(a+b)^0 = 1$

$(a+b)^1 = (a+b)$

$(a+b)^2 = a^2 + 2ab + b^2$

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$\vdots$

$(a+b)^{99} = ??$

$$\boxed{(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k}$$

Q.11 $n = 3$??

Sol.

$$(a+b)^3 = \sum_{k=0}^{3} \binom{3}{k} a^{3-k} b^k$$

$$= \binom{3}{0} a^{3-0} b^{0} + \binom{3}{1} a^{3-1} b^{1} + \binom{3}{2} a^{3-2} b^{2} + \binom{3}{3} a^{3-3} b^{3}$$

$$= 1 \cdot a^{3} \cdot 1 + 3 \cdot a^{2} \cdot b + \frac{3!}{2!\,(3-2)!} \cdot a \cdot b^{2} + 1 \cdot 1 \cdot b^{3}$$

$$= \underline{a^{3} + 3a^{2}b + 3ab^{2} + b^{3}} = (a+b)^{3} \checkmark$$

Q.2) $\underline{n = 4}$

Sol. $$(a+b)^{4} = \sum_{k=0}^{4} \binom{4}{k} a^{4-k} b^{k}$$

$$= \binom{4}{0} a^{4-0} b^{0} + \binom{4}{1} a^{4-1} b^{1} + \binom{4}{2} a^{4-2} b^{2} + \binom{4}{3} a^{4-3} b^{3} + \binom{4}{4} a^{4-4} b^{4}$$

$$= \underline{a^{4} + 4a^{3}b + 6a^{2}b^{2} + 4ab^{3} + b^{4}} = (a+b)^{4}$$
